1. Prove the Eckman-Hilton Theorem (lecture notes Theorem 3.5)

2. An *H-space* is a pointed space (X, x_0) equipped with a map $\mu\colon X \times X \to X$ such that $\mu(x_0, x_0) = x_0$ and the functions $f_L, f_R\colon X \to X$ given by $f_L(x) = \mu(x, x_0)$ and $f_L(x) = \mu(x_0, x)$ are homotopic to the identity map $\mathrm{id}_X\colon X \to X$ via homotopies preserving the basepoint.

a) Let (X, x_0) be an H-space. For maps $\omega, \tau\colon (I^n, \partial I^n) \to (X, x_0)$ define $\omega \odot \tau\colon (I^n, \partial I^n) \to (X, x_0)$ by $\omega \odot \tau(t) = \mu(\omega(t), \tau(t))$. Show that in $\pi_n(X, x_0)$ we have $[\omega \odot \tau] = [\omega] \cdot [\tau]$ where the multiplication on the right hand side is the usual multiplication in $\pi_n(X, x_0)$.

b) Show that if (X, x_0) is a path connected H-space then the action of $\pi_1(X, x_0)$ on all higher homotopy groups is trivial (i.e. the space X is simple).

3. Prove Proposition 3.7 from the lecture notes.

4. Let X be an n-dimensional CW complex such that $\pi_i(X) = 0$ for all $i \leq n$. Show that $X \simeq *$.

5. Let (X, A) be a pair with the homotopy extension property (Definition 2.13). Assume that we have a map $h\colon X \times \{0\} \cup A \times [0, 1] \to Y$ and that $H, H'\colon X \times [0, 1] \to Y$ are two extensions of h. Show that $H_1 \simeq H'_1$ (rel A).