1. Let (X, x_0), (Y, y_0) be path connected spaces such that $\pi_1(X, x_0) = 0$ and $\pi_n(Y, y_0) = 0$ for $n > 1$. Show that the space $Z = X \times Y$ is n-simple for all $n > 1$.

2. Show that if $p\colon E \to B$ is a Serre fibration and B is a path connected space then p is onto.

3. Given a Serre fibration $F \xrightarrow{i} E \xrightarrow{p} B$, use the homotopy lifting property to define an action of $\pi_1(E)$ on $\pi_n(F)$, that is a homomorphism from $\pi_1(E)$ to $\mathrm{Aut}(\pi_n(F))$ – the group of automorphisms of $\pi_n(F)$, such that the composition $\pi_1(F) \xrightarrow{i_*} \pi_1(E) \to \mathrm{Aut}(\pi_n(F))$ is the usual action of $\pi_1(F)$ on $\pi_n(F)$. As a consequence, if $i\colon F \to E$ is an inclusion of the fiber of a Serre fibration and $\pi_1(E) = 0$ then F must be a simple space.

4. Given maps $p\colon E \to B$ and $f\colon X \to B$ the *pullback* of p along f is the space $f^*E \subseteq X \times E$ given by

$$f^*E = \{(x, e) \in X \times E \mid f(x) = p(e)\}$$

Consider the map $p'\colon f^*E \to X$ given by $p'(x, e) = x$. Show that if p is a Hurewicz (or Serre) fibration then p' is a Hurewicz (or, respectively, Serre) fibration.

5. Let $p\colon E \to B$ be a Serre fibration. Show that if B is a path connected space then for any $b_0, b_1 \in B$ we have $\pi_n(p^{-1}(b_0)) \cong \pi_n(p^{-1}(b_1))$ for all $n \geq 0$.

Hint. Show this first assuming that B is a contractible space. In the case of a general Serre fibration $p\colon E \to B$, take a path $\omega\colon [0, 1] \to B$ such that $\omega(i) = b_i$ for $i = 0, 1$ and consider the fibration $p'\colon \omega^*E \to [0, 1]$ defined as in problem 4.